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NUMERICAL RECIPES FOR EQUIVALENT CONTINUUM MODELING OF JOINTED SYSTEMS

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Abstract. *Jointed rock mass presents a nonlinear compressible discontinuous mechanical system which is difficult to model using traditional continuum mechanics. Continuum models calibrated on a small scale rock samples are often applied in large scale engineering calculations. Yet, it is known that on the large scale the response of rock masses is drastically different and depends both on rock and joint properties. Large scale rock response is poorly understood because experimental studies are scarce and expensive as well as the computational efforts to study this response. We have developed tools to model such systems in a wide range of loading conditions which are based on advanced contact algorithms and hybrid FD/FE methods. The papers illustrates some of the ideas for meso-scale modeling which could be applied to build better continuum models for rock mechanic applications for specific conditions.*

1 INTRODUCTION

Advanced numerical methods to model dynamic processes in jointed rock are required for a number of problems of strategic importance such as, stability of underground structures to shock wave loading, monitoring of mining operations, discrimination of underground events etc. Most of engineering applications supporting this research rely on continuum models to describe mechanics of rock masses under dynamic loading. These models are typically calibrated at lab scale using rock samples from the site of interest. Yet, it is known that large scale response of rock masses is drastically different and depends both on rock and joint properties. Meso-scale numerical modeling can provide useful information to help build better continuum models for engineering applications. Jointed rock mass presents a nonlinear compressible discontinuous mechanical system which is difficult to model using traditional continuum mechanics. Therefore, development of hybrid discrete-continuum methods is important. Some aspects of the problems mentioned above can be better handled with the continuum mechanics while the others are better solved with discrete methods. For example, is important for defense community to evaluate risks to deeply berried structures (tunnels, bunkers, pipelines etc) to surface attacks. This type of problems typically include two time scales. The first one, is defined by the time to propagate the wave from the surface to the structure and the second one is defined by the collapse time under gravity. Hybrid discrete-continuum approaches are desirable to model different aspects of this problem where a continuum approach can be applied to model wave generation and wave propagation phases but a discrete approach is desirable to model the collapse of the structure.

Equivalent continuum response of jointed rock may be frequency dependent. This may be important for discrimination of underground events within the nuclear monitoring program which serves to discriminate between natural events such as earthquakes and underground cavity collapses from products of human activity (nuclear explosions, mine explosions etc). The challenging part in modeling of such problems is a large wave frequency scale that should be resolved (from 1 KHz around the source to less than 1 Hz to model seismic wave propagation at regional distances where the monitoring is performed. Two strategies can be used to solve such problems: continuous remap from smaller to large scale within one code or coupling different tools which are used in various frequency ranges. From continuum modeling point of view this class of problems presents another challenge. That is for each wave length the equivalent continuum may have a different mechanical characteristics (such as effective stiffness and yield strength) exhibiting scale dependence.

Another class of problems includes monitoring industrial underground operations such as, hydro-fracturing operations to boost gas production by enhancing fracture network, underground coal gasification and geothermal systems. The problems encountered in such operations are primarily quasi-static where the stress equilibrium is achieved at any time but is slowly changing due to various factors caused in part by the operation procedure. Equivalent continuum properties are required for such systems to evaluate the stability criteria and predict stress evolution during the operation. At the same time, since direct real time insitu stress measurements are limited and very expensive, seismic monitoring can be used in combination with the numerical simulation of microseismicity cased by the operation. The difficulty here is that one need to resolve vastly different time scales which coexist: the natural scale of evolving insitu stress due to tectonic motion (few years), the time scale of the operation (one day to a month)

and the characteristic time of microseismic events (< 1 sec and less).

Motivated by the problems and challenges described above, we are developing tools to model dynamic response of discontinuous systems in a wide range of loading conditions which are based on advanced contact algorithms and hybrid FD/FE methods. The papers illustrates some of the ideas for meso-scale modeling which could be applied to build better continuum models for rock mechanic applications.

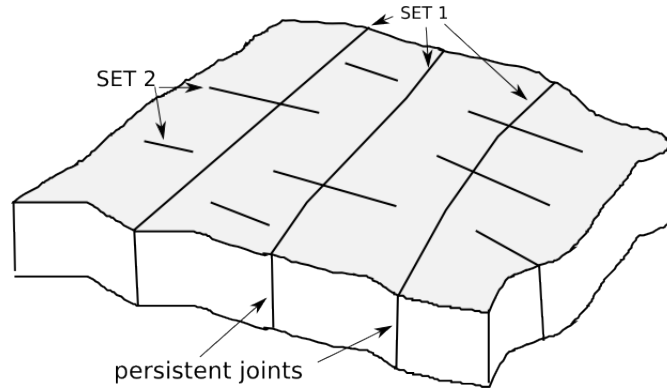


Figure 1: Schematic representation of joint sets

2 EQUIVALENT CONTINUUM MODELS FOR JOINTED ROCK MASSES

Since it is practically impossible to model every single joints and fault for a large scale problem, equivalent continuum models are often applied in rock engineering calculations. Most of these calculations assume that the rock mass can be treated as an elastic material. Effective elastic moduli are found as a combination of intact elastic moduli and elastic stiffnesses of the joints. The simplest approach used by Fossum [1] et al assumes that the joints are linear elastic layers of material randomly distributed in space. If the joint distribution density and aperture are known, then the effective elastic moduli can be found by averaging elastic compliances. This results in an isotropic elastic medium with reduced elastic moduli. Very little has been done to study response of jointed rock mass for the cases of large deformations when it cannot be considered linear anymore. Nonlinear compressibility at low confinements comes primarily from the nonlinear character of the joints [5]. A number of constitutive models has been developed to model shock wave propagation in jointed rock caused by explosions. Most of them are completely unrelated to the properties of the intact rock samples measured at the lab except that they apply the same form of constitutive equations. Model parameters are then fitted in numerical simulations to match observed large scale response. An attempt to build large scale model based on the extension of the small scale model was made in [6], where effect of joints on overall compressibility and strength has been studied for quasi-static loading conditions. It helped to develop scaling rules for jointed rock masses. Dynamic testing of synthetic rock mass was studied in [7].

3 MESO-SCALE MODELING OF JOINTED ROCK RESPONSE TO DYNAMIC LOADING

We will consider dynamic loading of rock masses to the stresses above the strength limit of intact material. In this case responses of both joints and intact blocks are inherently nonlinear, which makes the task of determining of effective properties much more difficult. Strength properties of geologic materials are known to vary with the sample size, therefore they may depend on the size of the representative volume defined by the wave length in dynamic problems. It means that the equivalent response can be frequency dependent. Damage in jointed rock mass tends to localize at the joints which are much thinner than the rock blocks which they connect. To resolve the joints special numerical techniques (such as thin elements [2], contact elements etc) are required. Joints are often represented in joint sets which are not necessarily persistent. Schematic representation of joint sets is shown in 1. Meshing non persistent joint sets is very difficult. As damage develops in the rock mass, material transitions from continuum medium with singularities (joints cracks etc) to a granular medium. Modeling such transition algorithmically difficult. Taking into consideration all these difficulties we have developed a few recipes to model these systems using discrete-continuum approach. The medium is presented as an assembly of sudiscretized meshed blocks connected by the contact elements. Nonlinear Finite Difference and Finite Element solvers are used to model deformations in the blocks and simple common plane contact elements described in [8] are used to model interaction between the blocks. In addition, we consider fragmentation of the blocks into a number of sub-blocks with possibility of dynamic mesh refinement withing the blocks.

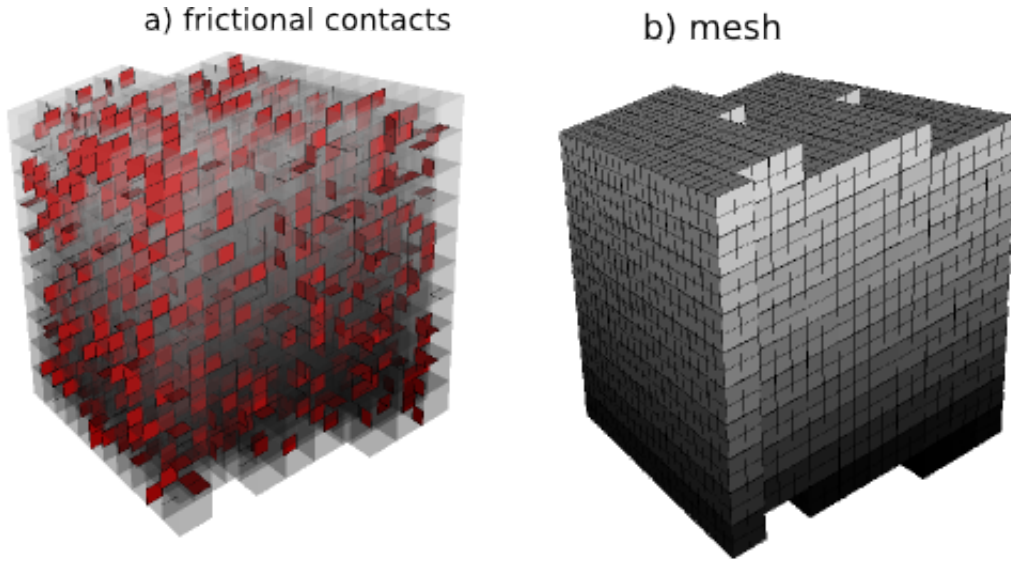


Figure 2: Using variable cohesive contacts to mesh 25% persistent joints: a) frictional contacts, b) the mesh built using stacked parallelepiped blocks mesh.

3.1 Recipe #1: Modeling non-persistent joints using variable frictional properties at the contacts

Since meshing non-persistent joints is a nontrivial problem, we suggest the following simpler way to model joint sets. First, we generate a persistent joint sets which present a system of parallel planes (lines in 2D) ,for example, three-joint system which is very common in nature.

Such planes will cut parallelepiped blocks which could be easily meshed using hexahedral elements. Secondly, we set-up elastically stiff contacts with higher cohesive properties which will effectively stop joints both from sliding and normal compression mimicking a tied boundary that way. Figure 2 shows an example of the meshing of such three joint system. The joints were 25% persistent in two directions and 50% persistent in one direction. Only frictional joints are shown as well as the mesh used. Since blocks represent independently meshed volumes, adaptive meshing can be applied using variable meshing accuracy for the blocks. One can present continuum as an assembly of the blocks tied together through cohesive contacts. Figure 3 shows a 2D example, how by varying the joint properties one can transition from a continuum to a jointed medium. We set all joints as stiff cohesive contacts and used velocity

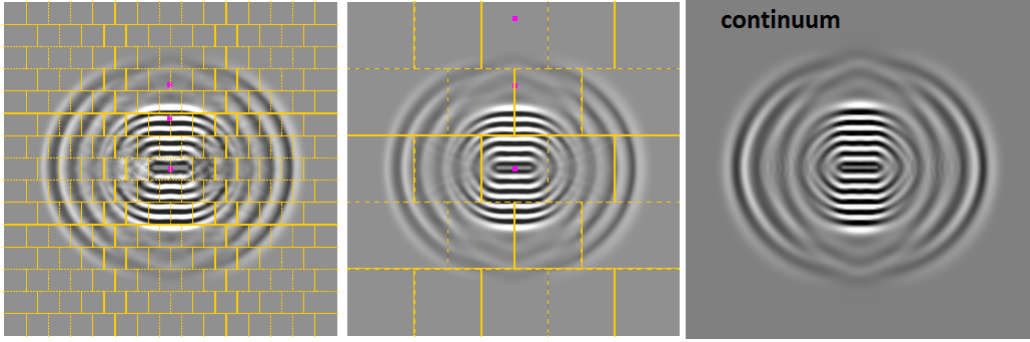


Figure 3: Comparing wave propagation in a system with locked joints with continuum calculation. 15X15 blocks is on the left, 5x5 blocks is in the middle and continuum is on the right. Velocity range -5 m/s (dark) to 5 m/s (light), time=60 ms

boundary in the middle of the mesh to generate the waves. A group of nodes in the middle of the region was controlled by a velocity boundary moving horizontally as it is shown in Fig.4.

As it is seen from the Figure 3, there is very little difference between the continuum calculations without joints (on the right) and the region consisting of various number of blocks connected by joints as long as the joints are strong.

Figure 4 shows the contours of horizontal velocity calculated at two target locations at the same range from the source. The cohesion on the joints was reduced so the joints were allowed to slip. Reduced cohesion also affected the tensile strength of the joints. This caused the joints on the left side to open and as a result of it the wave propagated asymmetrically.

3.2 Recipe #2: Efficient hybrid contact search algorithm

Presence of multiple contacts can make calculations computationally expensive. During dynamic loading of jointed systems only a small number of contact surfaces remain active at any time. Therefore, most of contacts do not change connectivity over many computational cycles. Because of that, they can be often excluded from the contact search. Thus, contact faces should only be added to the list of candidates which can create new contacts if there is a significant shear displacement taking place at these faces. We have introduced two types of contacts: cohesive contacts and collisional contacts which are initially defined by the boundary conditions set at the faces. The collisional contacts (called here type 0 contacts) are assumed to change dynamically every few cycles and are typically assigned to the external boundaries of discrete blocks (such as, for example sand grains or moving objects). The cohesive contacts in turn (type 1) are assumed to stay in place until a certain amount of shear slip is accumulated

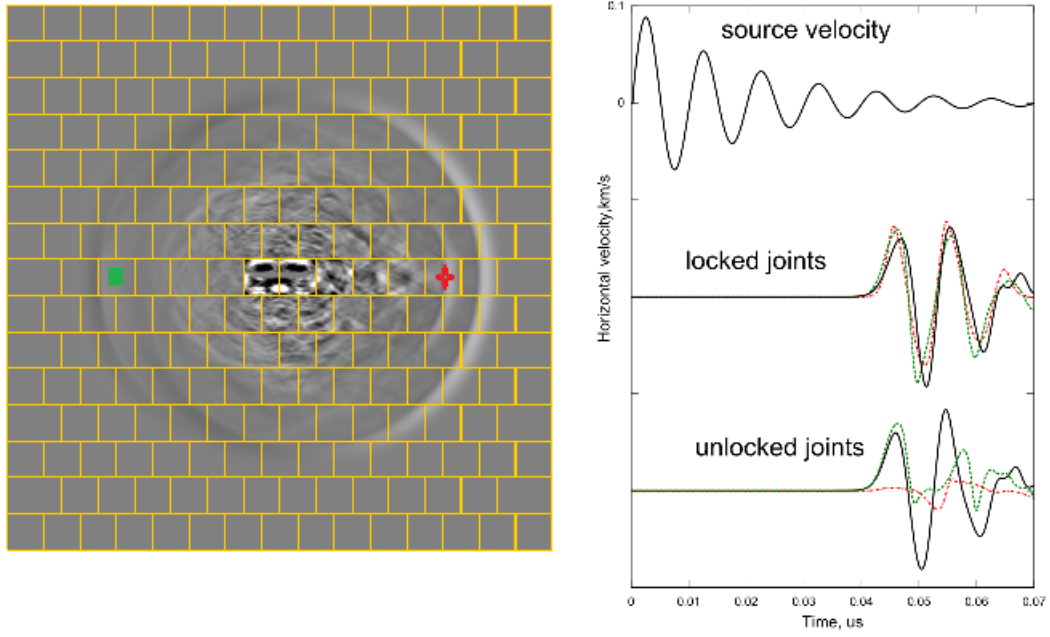


Figure 4: Wave propagation in a system of 15x15 blocks with unlocked joints

at the faces. When it happens, the contact can be considered broken and its type is switched to type 0 (collisional contacts). All faces of type 1 contact are excluded from the contact search once the contacts involving those faces are established. The contacts of type 0 are generally included into new contact search. To improve the computational efficiency further, they can be temporarily excluded from the new contact search if their velocities are small, which means that they do not have a chance to move and create a new contacts. Figure 5 shows an example of using hybrid contact algorithm for shear loading of a representative region in 2D. A jointed system with two sets of joints was loading using velocity boundary conditions (shown with black dots) applied to the nodes of the elements on the periphery of the region. Plastic slip has developed overtime at the contacts close to the boundaries and in the middle of the region which triggered the transition from type 1 contact to type 0 contact. The problem runs roughly two times faster when the hybrid contacts are used.

Figure 6 shows problem set-up for calculation of spherical explosion in a 3 set jointed rock formation. The region was meshes as an assembly of paralelipiped blocks subdiscretize into different level of accuracy. The source block was meshed differently using conforming hexahedral meshes for a sphere and a block with a spherical cavity of the same size. The radius of the was 0.625 m, and the size of the source block was 4 times bigger than the other blocks. The source was modeled as an ideal gas material with density of 1.32 g/cc, specific internal energy of 3.9 kJ/g and gamma parameter of 1.3. Cohesive contacts (type 1) were used between the block which later transitioned into collisional contacts (type 0) as it is shown in Figure 7, which also shows the pressure contours in the range (0.001-0.01 GPa).

3.3 Recipe #3: Using discrete meshes to model fracture and fragmentation

To model damage and fragmentation in rock masses one can either use a continuum plasticity model which includes some history variables describing the strength softening or allow an explicit description of new fractures in the material at localized zones of failure which can be

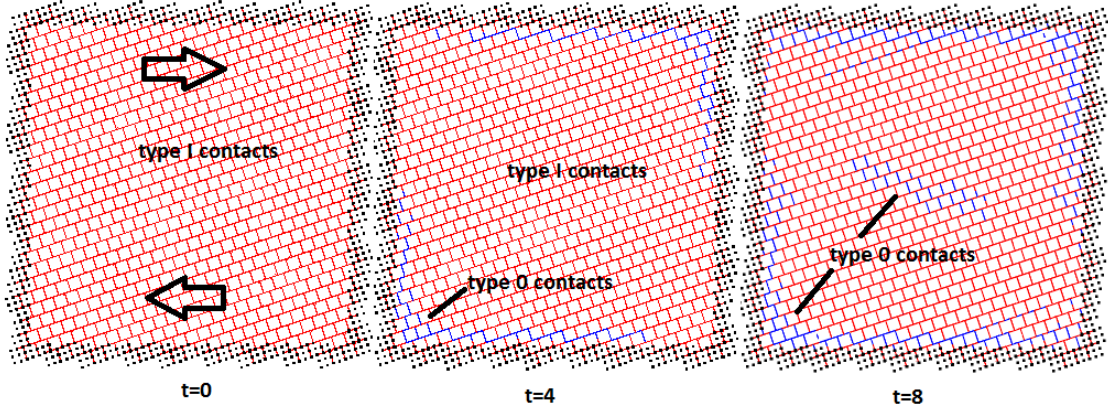


Figure 5: Shear loading of an RVE with two sets of joints using hybrid contact

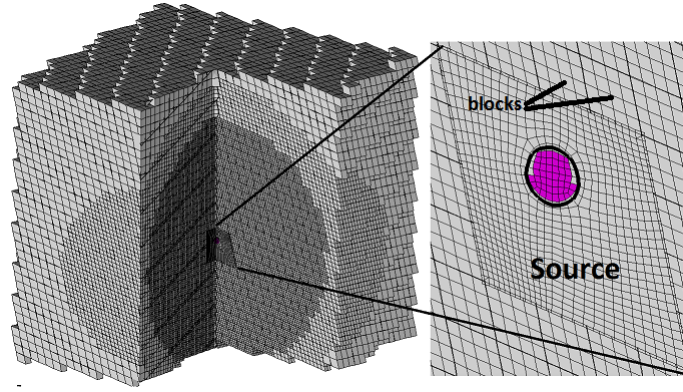
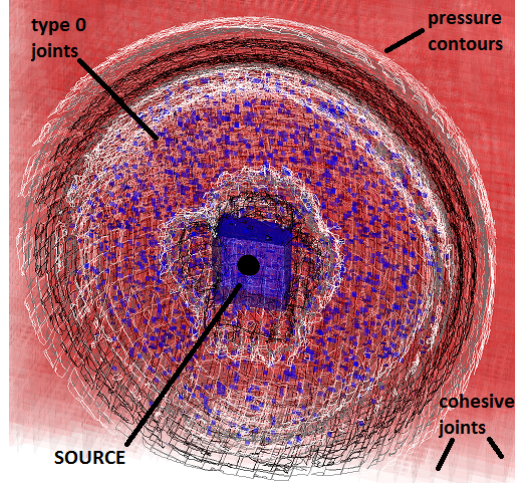


Figure 6: Mesh and block locations for 3D explosion in a jointed rock

significantly smaller than the cell size. The first, continuum approach, can be mesh sensitive, since sub-cell localization zones cannot be accurately resolved. The second approach includes algorithmic difficulties to transition from continuum to discrete description of the failure zone. We believe that when new localized failure zones are created (such as cracks, joints) mechanical properties in those zones (such as friction, dilatancy, stiffness) are significantly different from the intact material, and, therefore, it is difficult to describe them using the same strength model as the one used for the intact material. Even if such a wide-range strength continuum model is designed one should resolve the localization zone which could be few orders of magnitude less than the characteristic size (cell size). Therefore, the second, discrete, approach looks more attractive to us since it applies a special modeling for the localized zones of failed material in addition to the continuum modeling elsewhere. We hope that such a discrete-continuum approach can be useful in meso-scale modeling of fracture and fragmentation especially in the systems with preexisting discontinuities. We apply common plane contact [7] with history variables describing damage at the contacts which in the limit of stiff and strong contacts describe continuum (as it was shown above) and, once material breaks, transition to frictionless slide contacts. Contacts change type from type 1 to type 0 (collisional contacts) when such transition takes place. Figure 8 shows an example of shear loading of an RVE made of tightly packed polygonal blocks. The blocks were meshed using premeshed spheres which were placed inside the polygons and deformed radially to conform to the boundaries. Then an equipotential zon-

Figure 7: Pressure contours and joint types at time $t=3$ ms

ing was applied to improve the mesh inside the polygone. Velocity controlled boundaries were applied at the boundary nodes.

Such method limits the possible path for the fracture surfaces to the existing mesh lines which may create a mesh bias. Possible solution is to seed the centers of weakness in the material and use meshing technique which connects those centers (for example, Voronoi triangulations). Changing the mesh resolution in this case will guarantee that all meshes offer the same fracture paths aligned with the material weakness and , thus, less likely will be mesh sensitive.

Figure 10 shows dynamic block fragmentation.

3.4 Recipe #4: Using dynamic decoupling with mesh refinement

The alternative to a prefactured mesh described above is a dynamic element decoupling. It has an advantage if only small fraction of the material experiences failure during the loading. We have implemented the following decoupling algorithm: Once the shear or tensile stress in an element reaches a critical value (within 10 percent to the failure criterion) this element is decoupled from the mesh and contact faces created for the existing faces of the element as well as new external faces which belong to the neighboring elements. The contact variables are initialized immediately to support both the normal and the shear stresses interpolated to the interface from the adjacent element centers. If high resolution is required to resolve the fracture path withing the element, the element can be subdiscretized or remeshed. In this case, more than one external face is created for each side of the element which now becomes a subdiscretized mesh block. Figure 9 shows a simple 2D problem illustrating the dynamic decoupling algorithm with block refinement. A cylindrical tunnel in an imbricate wall is impacted by a steel cylindrical projectile with 100 m/s velocity.

4 CONCLUSIONS

In this paper we have discussed outstanding problems in modeling mechanics of heavily jointed rock masses under dynamic loading conditions. We have offered some numerical recipes which may help to overcome difficulties in modeling of large-scale responses of jointed rock masses. The brief list of these recipes is given below.

- We advocate discrete-continuum approach, where discrete features of the rock mass is

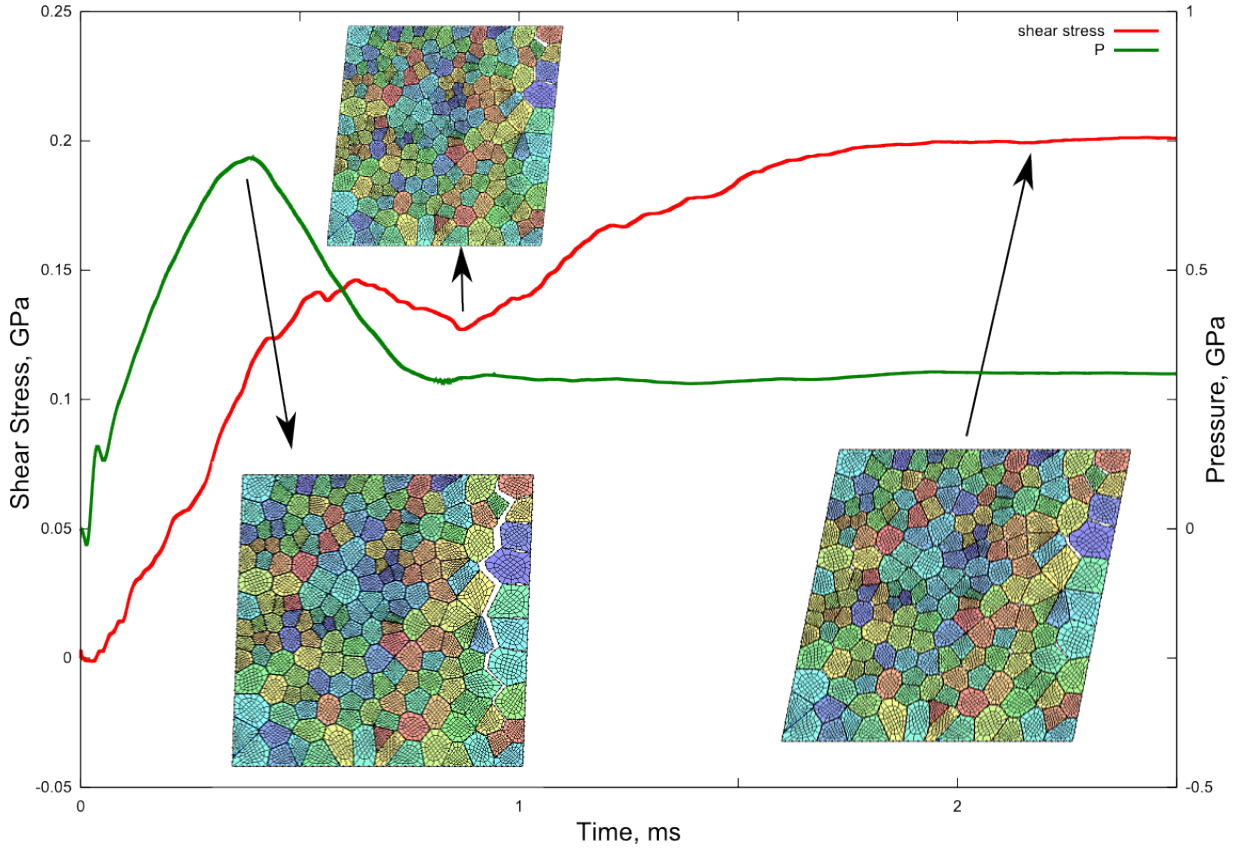


Figure 8: Shear stress and pressure evolution for 2D RVE made of packed blocks. Block deformations is shown at various times

explicitly modeled only where the highest fidelity is required (for example, around the structures) and everywhere else they are replaced by equivalent continuum models. The equivalent continuum models can be derived for specific sites and specific loading conditions by gradually replacing areas where discrete model is applied with a continuum model where the model parameters can be optimized to produce minimal deviation between the two models.

- To calibrate the continuum model meso-scale calculations can be performed on Representative Volumes where the joints are represented by contacts with history variables.
- Initial properties of these contacts can vary to model transitions between indefinitely strong cohesion in the limit of continuum and a frictional surface where material breaks along the contact surface. Thus by using initially strong contacts we can simplify meshing task for non-persistent joints.
- Since material on both sides of the strong contact stays together until it breaks, there no need to apply contact search algorithm for those contacts. Thus, we introduce two types of contacts: the cohesive contact type, not used in contact search, and the collisional contact type which represents moving surfaces that are used to find new contacts. When many contacts are used to model both existing and potential fracture surfaces, this separating is crucial to drastically reduce the time of computations.

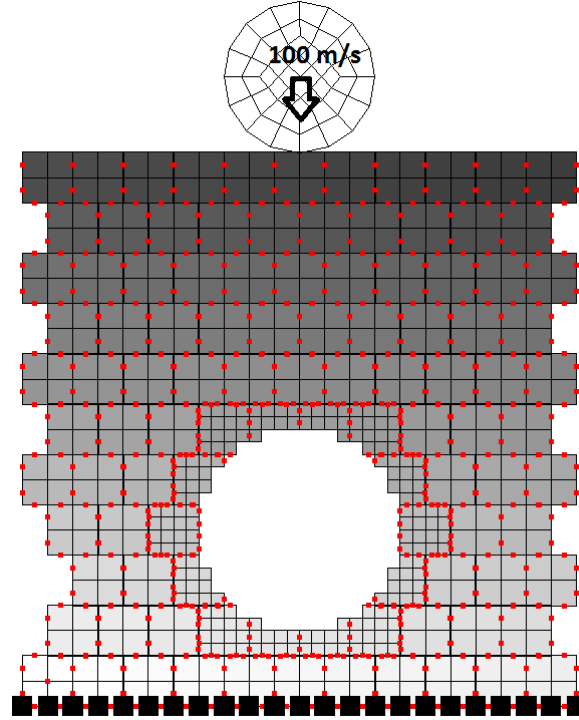


Figure 9: Cavity collapse under impact: Meshed blocks, contact boundaries between blocks (small dots) and fixed bottom boundary (large dots)

- Mesh sensitivity is a known problem for continuum modeling of materials with damage. One way to defeat it is to introduce a time or space scale into continuum model. To match realistic response very often this scale turns out to be much smaller than the element size which makes calculations impractical. We suggest to model softening response only at the contact surfaces. The characteristic size in this case is defined by the joint aperture, which is typically two orders of magnitude less than the element size. Assuming that the stability time step is not defined by the contact model, we can converge to mesh independent solution without making the time step dependent on the characteristic size. The limitation of our approach is that the damage may only happen along the mesh lines where the contacts exist.

5 ACKNOWLEDGMENT

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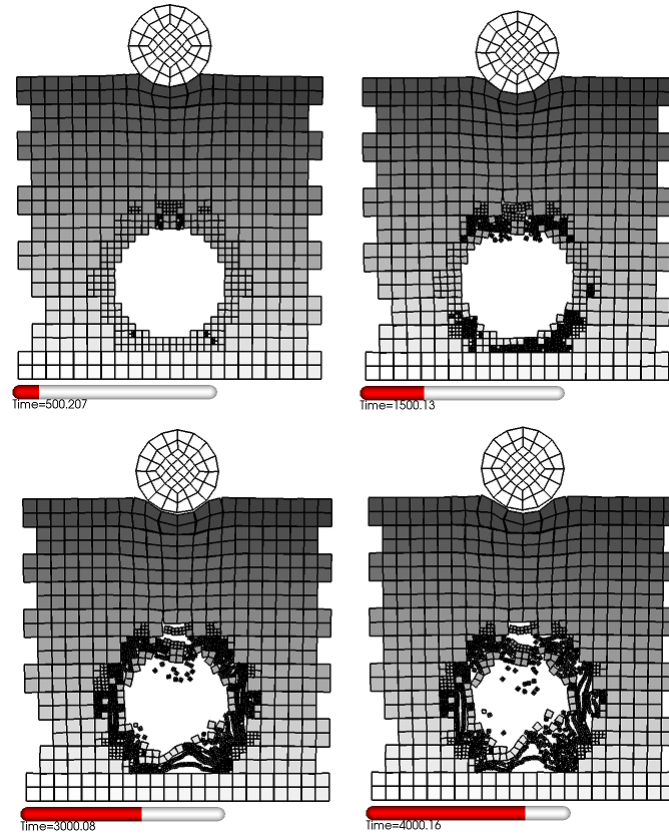


Figure 10: Cavity collapse under impact: block fracture and motion around the cavity at different times

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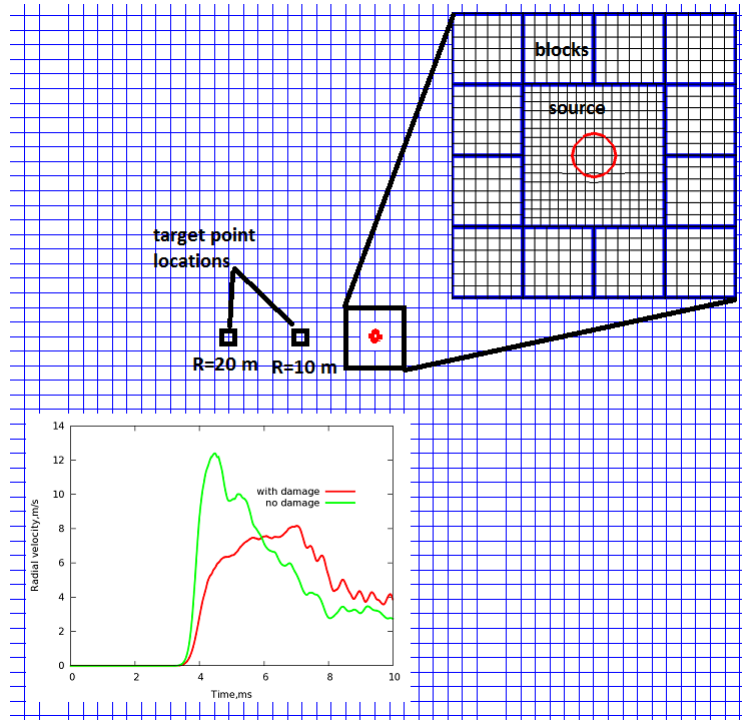


Figure 11: Block boundaries, target point location and computational mesh for 2D explosion in jointed rock. Radial velocity evolution at range 20 m is shown in the low left corner

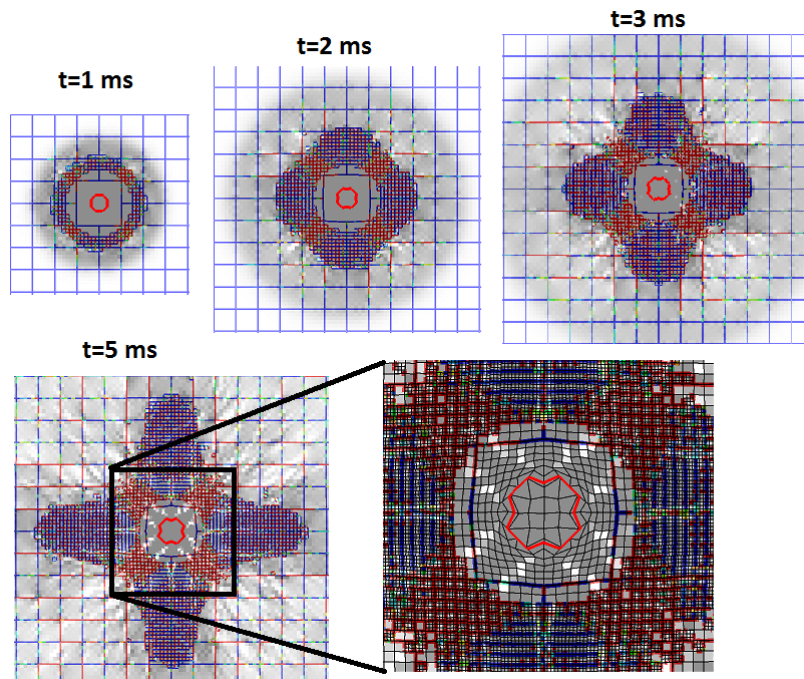


Figure 12: Pressure contours (0-0.1 GPa), shear slip at the joints (0-1) and material boundary between the source and rock. Enlarged source region is shown on the right